Thermal-response time of superconducting transition-edge microcalorimeters

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We investigate limits on the thermal-response time of superconducting transition-edge microcalorimeters. For operation at 0.1 K, we show that the lower limit on the response time of a superconducting transition-edge microcalorimeter is of order 1 μs due to the heat diffusion time, electrical instabilities, the amplifier noise, and the critical current of the superconducting film. The response time is not limited by self-heating effects and is independent of the intended photon energy. However, design constraints associated with the inductance of the bias circuit make it difficult to achieve the fastest response times for devices with heat capacities high enough for x-ray and gamma-ray detection. [S0021-8979(98)07308-3]

I. INTRODUCTION

Superconducting transition-edge sensor (TES) microcalorimeters are promising spectroscopic detectors for photons from the infrared to gamma rays. Excellent energy resolution has been achieved with these devices. However, applications such as x-ray microanalysis, optical astronomy, and astronomical observations of bright x-ray sources require faster response times and count rates than have been achieved to date without sacrificing energy resolution. While the theoretically achievable energy resolution has been well studied,[1,2] the theoretically achievable response time has received less attention. In this article we discuss the lower limits on the response time imposed by the physics of the TES and by the bias circuit.

A microcalorimeter consists of a thermometer, an absorber with a heat capacity C, and a thermal conductance G to a heat bath. When a photon interacts with the absorber, the deposited energy is thermalized and measured as a temperature rise. After the event, the temperature of the thermometer returns to the equilibrium temperature with a response time τ. Microcalorimeters are typically operated at low temperatures ($T \sim 0.1$ K) where materials have small heat capacities (leading to a large temperature rise), thermal noise sources are small, and sensitive thermometers are available. Most microcalorimeters have been fabricated using semiconductor-thermistor thermometers read out with field-effect transistors.[1] The relatively low sensitivity of a thermistor requires the use of an absorber with a low specific heat such as semiconductors, semimetals and superconductors.

TES thermometers are superconducting thin films biased within the transition from the superconducting to the normal state. In this region the resistance of the film is extremely sensitive to the temperature. TES thermometers are leading candidates for use in fast microcalorimeters since their sensitivity allows the use of normal-metal absorbers even though normal metals have a high specific heat and consequently small temperature changes. Normal-metal absorbers are advantageous because they thermalize and uniformly diffuse energy more quickly and completely than semiconductors, semimetals, and superconductors.[3,4]

TES microcalorimeters have achieved an energy resolution of 7.2 ± 0.4 eV full width at half maximum (FWHM) for 6 keV x rays with a response time of approximately 200 μs.[5] TES microcalorimeters designed to operate at optical photon energies have achieved an energy resolution of 0.2 eV FWHM for 4 eV heat pulses, with a response time of 8 μs.[6] We show in this article that the best achievable response time for operation at 0.1 K is of order 1 μs and is in principle independent of photon energy, although the design constraints are more challenging at higher energies.

We begin by discussing the bias circuit for TES microcalorimeters and the effect of negative electrothermal feedback. We then consider the factors which constrain the response time of TES microcalorimeters, including thermalization and diffusion times, the requirement of electrical stability, amplifier noise, the critical current of the superconductor, and self-heating effects.

II. TES BIAS AND ELECTROTHERMAL FEEDBACK

In order to discuss constraints on the response time of TES microcalorimeters, we must first discuss the TES bias circuit. The bias source used with a TES microcalorimeter can be anything from a perfect current bias to a perfect voltage bias. Throughout this article we assume a perfect voltage source. As will be shown, a voltage source leads to a faster detector response than high impedance sources and yields a minimum response time close to the heat diffusion time. We consider a circuit in which a voltage bias is applied to a series arrangement of the TES and the input coil of a superconducting quantum interference device (SQUID) amplifier which is used to measure the current through the sensor. When a photon is absorbed by the microcalorimeter, the temperature of the TES increases, its resistance increases, the current flowing through it decreases, and this decrease is sensed by the SQUID. Recent developments such as series-

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array dc SQUIDs allow operation at 10–100 MHz bandwidth,[7] making fast response times possible.

The bias circuit directly influences the response time of TES microcalorimeters through electrothermal feedback. Negative electrothermal feedback results when the TES is voltage biased and the heat bath is cooled to well below the transition temperature. Electrothermal feedback causes the film to self-regulate its temperature within the region of rapid resistance change associated with the transition.[2,8] Self-regulation results because as the film cools, its resistance drops, and the Joule heating, V^2/R, increases. Joule heating thus provides negative feedback which tends to raise the temperature, and a stable equilibrium is established when Joule heating matches the heat loss into the bath. Self-regulation can occur with a characteristic time constant much shorter than the natural C/G time constant. For a voltage-biased sensor, the effective response time is[9]

$$\tau_{\text{eff}}=\frac{C/G}{1+\alpha\phi/n}. \tag{1}$$

Here $\phi\equiv 1-(T_b/T_0)^n$ is a measure of how far the detector is biased above the bath temperature, T_b is the temperature of the heat bath, T_0 is the equilibrium temperature of the sensor, the logarithmic sensitivity $\alpha\equiv d\ \log R/d\log T$ at constant V is a unitless measure of the sharpness of the superconducting transition, and the thermal-conductance exponent n is a number (typically about 4) that depends on the dominant thermal impedance between the heat bath and the electrons in the superconducting film. Since α can be of order 1000 for transition-edge thermometers, the effective response time can be two orders of magnitude shorter than the C/G time constant. Throughout this paper, operation in the strong-feedback limit will be assumed, where $\alpha\gg n/\phi$.

III. THERMALIZATION AND DIFFUSION

In this section, we discuss the limitations placed on the response time of a TES microcalorimeter by the rate of thermalization of heat in the absorber and diffusion of heat within the absorber and TES. The thermalization time is the time required for energy deposited in the absorber to relax to a Fermi–Dirac distribution in the electron system. The diffusion time is the time required for heat in the electron system to diffuse spatially. As will be discussed, there are different constraints on the diffusion times in the absorber and the TES thermometer.

We first consider the effect of the thermalization time on the TES microcalorimeter response time. When a high-energy photon interacts in a normal-metal absorber, it ejects a high-energy inner-core electron which then forms a local hot spot by electron–electron and electron–phonon interactions. At low temperatures, inelastic electron–electron interactions are thought to dominate the thermalization process. Unfortunately, inelastic rates in polycrystalline metal films at low temperature ($T\sim 0.1$ K) and low sheet resistance ($T<1\ \Omega$) are not fully understood. Experimental values for electron–electron interaction times (and thus thermalization times) under these conditions tend to be nanoseconds, significantly faster than theoretical computations.[10–12] It will be assumed in this article that thermalization times are fast compared to other time scales in the detector (such as heat diffusion time in the thermometer and C/G times) and can be neglected.

The diffusion times of the TES microcalorimeter can limit the response time. The diffusion time in both the absorber and the microcalorimeter are important. We consider the diffusion time in the absorber first. If the diffusion time in the absorber is too slow, variations in the interaction position of the photon in the absorber will cause variations in the shape of the leading edge of the pulse. Fortunately, the diffusion time in the absorber can be made extremely fast. Diffusion times are dependent on both the material and the geometry; in the general case they must be computed numerically. For a simple geometry, an estimate of the diffusion time can be made from the diffusivity, $D=v_f l$, where v_f is the Fermi velocity and l is the electron mean-free path of the metal. The characteristic diffusion time associated with a length scale L is $\tau_{\text{diff}}=L^2/D$. For example, an absorber for an x-ray microcalorimeter might consist of a gold film with dimensions 250 μm$\times$250 μm$\times$5 μm, for which a mean-free path of about 2 μm would be expected in a high quality film. With these absorber parameters, the calculated diffusion time in the absorber is about 22 ns. A detector thermal response time of order 1 μs should thus be achievable without significant variations in pulse shape. It should be noted that even if a detector has significant pulse shape variation due to incident photon position, it may be possible to deconvolute the position of the interaction from the energy so as not to sacrifice energy resolution. This procedure can be as simple as correcting the pulse energy with a rise time measurement, or it may require a more complicated multivariate fit.

The heat diffusion time in a TES thermometer can also limit the detector response time. Unfortunately, it is generally not possible to make the diffusion times in the thermometer as fast as diffusion times in the absorber due to engineering constraints on the resistance of the TES. Although slow diffusion times in the thermometer do not lead to pulse shape variations, they can nonetheless lead to significant degradation in energy resolution. Because information is lost at frequencies where thermalization and diffusion take place, these processes must occur at frequencies higher than the bandwidth ΔB of the filter function used in pulse processing. The bandwidth necessary to approach the optimal energy resolution depends upon the frequency spectrum of the signal and noise, which are determined by the response time of the microcalorimeter and the effect of electrothermal feedback. We first consider the required bandwidth of the filter function and then discuss the limitations placed on the response time of the microcalorimeter by the diffusion time in the thermometer.

In order to discuss the required bandwidth of the filter, it is necessary to consider the sources of noise in the sensor. The fundamental noise sources in a TES microcalorimeter are the phonon noise (thermodynamic power fluctuations) of the thermal conductance G connecting the microcalorimeter to the heat bath and the Johnson current noise of the TES. Although both of these noise sources are strongly frequency

dependent, in the strong-feedback limit their quadrature sum depends only weakly on frequency. In particular, the power spectral density of the quadrature sum of Johnson and phonon noise of the TES in the strong-feedback limit is[2]

$$S_I = \frac{4kT_0}{R_0}\,\frac{n/2+\omega^2\tau_{\rm eff}^2}{1+\omega^2\tau_{\rm eff}^2}, \tag{2}$$

where R_0 is the resistance of the sensor at equilibrium and ω is the angular frequency. Since typically $n\sim 4$, we may assume to a rough approximation that the noise in the sensor is white.

In the presence of white noise, the Wiener optimal filter has the same bandwidth as the signal,[1] and the required filter bandwidth is $\Delta B \approx 1/(2\pi\tau_{\rm eff})$. Diffusion in the TES must occur at frequencies higher than the required filter bandwidth to avoid degradation in energy resolution, so the characteristic diffusion time in the thermometer must be somewhat shorter than $\tau_{\rm eff}$. However, this result is only valid in the strong-feedback limit. When little electrothermal feedback is used, the quadrature sum of the noise sources have a strong frequency dependence. In this case, the required filter bandwidth can be hundreds of times larger than the bandwidth of the signal, and the diffusion time in the TES must be hundreds of times shorter than $\tau_{\rm eff}$.[3] Strong negative electrothermal feedback must be used for the response time to approach the diffusion time in the TES.[2]

While diffusion times in the TES cannot in general be made as fast as diffusion times in the absorber, it is possible to design thermometers with diffusion times faster than 1 μs by making the area small enough and the film mean-free path long enough. For example, choosing reasonable detector parameters, $v_f = 10^6$ m/s, $l = 100$ nm, and a diffusion length $L = 100$ μm, we arrive at a characteristic diffusion time in the sensor of $\tau_{\rm diff} \approx 0.1$ μs. The detector response time can thus be of order 1 μs without sacrificing energy resolution. Note that such a sensor would have a normal resistance of about 30 mΩ per square, so a low circuit inductance would be required for fast operation, as will be discussed in following sections.

IV. ELECTRICAL STABILITY

Another constraint on the response time of a TES microcalorimeter is imposed by the electrical bias circuit. If the inductance in the bias circuit is too large, a phase shift in the electrically-induced thermal response of the sensor can lead to oscillations.[9] In this section we describe the electrical bias circuit, calculate the effective electrical impedance of the TES, use this impedance to determine the condition for electrical stability, and discuss the constraint this condition imposes on the response time of TES microcalorimeters for a given circuit inductance.

The thermal response of the TES can by modeled by an effective electrical impedance $Z(\omega)$ which is independent of the bias circuit.[13] The differential equation governing the temperature of the transition-edge sensor is

$$C\,\frac{dT}{dt} = IV - K(T^n - T_b^n), \tag{3}$$

where I is the current flowing through the sensor, V is the electrical potential across the sensor, K is a material and geometry dependent parameter, T is the temperature of the sensor, and T_b is the temperature of the heat bath. The first term on the right-hand side of Eq. (3) represents the Joule power dissipation in the sensor and the second term describes the flow of heat to the heat bath. In the limit that T_b is close to T, the last term reduces to the familiar form $-G\Delta T$, where the differential thermal conductance $G \equiv dP/dT = nKT^{n-1}$, and $\Delta T \equiv T - T_b$. In formulating Eq. (3), we assume that all the equilibration times within the absorber and thermometer are fast compared to the global thermal response time and that self-heating leads to negligible temperature gradients in the device, so the absorber-thermometer system is at a uniform temperature T. If this condition were not true, the full thermal circuit model would have to be used, and the conditions for stability would be of higher order than quadratic.

In order to derive an effective electrical impedance, we expand Eq. (3) to first order around an equilibrium temperature and consider one Fourier component ω,

$$i\omega C T(\omega) = I_0 V(\omega) + V_0 I(\omega) - G T(\omega), \tag{4}$$

where I_0 and V_0 are the equilibrium current and voltage. As will be discussed later, the resistance of the TES, $R \equiv V/I$, can be a hysteretic function of temperature, applied current, and applied magnetic field. For the purposes of this simple calculation, we assume a low-current and low-field limit, and treat the resistance as a nonhysteretic function of temperature, $R = R(T)$. In this case,

$$V(\omega) = I_0 R(\omega) + R_0 I(\omega) = \frac{\alpha V_0 T(\omega)}{T_0} + R_0 I(\omega). \tag{5}$$

Combining Eqs. (4) and (5) and setting $\omega = 0$, we arrive at the zero frequency impedance

$$Z_0 \equiv Z(0) = \frac{V(0)}{I(0)} = -R_0\,\frac{\phi + n/\alpha}{\phi - n/\alpha}, \tag{6}$$

where, as previously, $\phi \equiv 1 - (T_b/T_0)^n$. The frequency dependent electrical impedance of the TES is[13]

$$Z(\omega) = Z_0\,\frac{1 + i\omega\tau(Z_0 + R_0)/2Z_0}{1 + i\omega\tau(Z_0 + R_0)/2R_0}. \tag{7}$$

The total impedance seen by the voltage source is the sum of the internal impedance of the voltage source, the effective electrical impedance of the TES, the inductance of the input coil of the SQUID, and any stray impedance. In practice, the internal impedance of the voltage source can be made very small, so that only stray inductances and the inductance of the input coil are important. The total impedance seen by the voltage source is then

$$Z_{\rm tot}(\omega) = Z_0\,\frac{1 + i\omega\tau(Z_0 + R_0)/2Z_0}{1 + i\omega\tau(Z_0 + R_0)/2R_0} + i\omega L, \tag{8}$$

where $L = L_{\rm SQUID} + L_{\rm stray}$.

We now consider the condition for electrical stability. If the zeros of $Z_{\rm tot}(\omega)$ are complex, the device response is oscillatory. If the zeros are in the right half of the complex

plane, the oscillations grow, whereas in the left they are damped. When the zeros come together on the real axis, the oscillations are critically damped. In practice, even damped oscillations present difficulties in device operation and pulse processing, and TES microcalorimeters are usually operated in a critically damped regime. We present here the critically damped solution.

The zeros of Eq. (8) come together on the real axis when

$$\frac{\tau}{\tau_{\rm elec}}=\frac{C/G}{L/R}=\frac{4Z_0-2R_0-4\sqrt{Z_0(Z_0-R_0)}}{R_0+Z_0}, \tag{9}$$

where $\tau_{\rm elec}\equiv L/R$ is the electrical response time of the sensor. Substituting Eq. (6) into Eq. (9), we arrive at

$$\frac{\tau}{\tau_{\rm elec}}=3\phi\,\frac{\alpha}{n}+1+2\sqrt{2}\,\sqrt{\left(\phi\,\frac{\alpha}{n}+1\right)\phi\,\frac{\alpha}{n}}. \tag{10}$$

This condition imposes a constraint on the response time of TES microcalorimeters. In the strong-feedback limit, we take $\alpha\gg n$ and $\phi\approx 1$ so that Eq. (10) reduces to

$$\frac{\tau n/\alpha}{\tau_{\rm elect}}=\frac{\tau_{\rm eff}}{\tau_{\rm elec}}=3+2\sqrt{2}\approx 5.8 \tag{11}$$

so that in the strong-feedback limit, the critical damping of oscillations requires that the effective response time be at least 5.8 times longer than the time constant L/R. Thus, the response time is dependent on the inductance and resistance of the electrical circuit.

V. AMPLIFIER NOISE

In this section we discuss the constraint that the amplifier noise places on the L/R time constant. In order to approach the optimal energy resolution, it is important that the current noise of the SQUID referred to its input be less than the current noise of the TES; if the SQUID noise is not smaller, the energy resolution of the detector will be degraded. We consider the current noise of the TES and SQUID, develop a constraint on the L/R time constant, and finally estimate the achievable electrical response time for a stable TES microcalorimeter with near-optimal energy resolution.

According to the theory,[1] the total TES noise is due to the phonon current noise in the sensor added in quadrature with the electrical Johnson current noise of the sensor. In the strong-feedback limit, the theoretical current noise of the sensor is dominated by the phonon noise at frequencies below the thermal response frequency. The current noise power spectral density (PSD) below the thermal knee frequency is approximately white, with value[2]

$$S_{I(\rm TES)}\approx\frac{4k_BT}{R_0}\,\frac{n}{2}, \tag{12}$$

where k_B is Boltzmann's constant and n is the thermal-conductance exponent.

The input inductance $L_{\rm SQUID}$ of the SQUID current amplifier can by adjusted by changing the number of turns of the input coil. The current noise referred to the SQUID input is a function of the input coil inductance and is typically parametrized by a coupled energy sensitivity of some multiple ξ of Planck's constant $L_{\rm SQUID}S_{I(\rm SQUID)}/2=\xi h$, so that

$$S_{I(\rm SQUID)}=\frac{2\xi h}{L_{\rm SQUID}}. \tag{13}$$

For typical SQUIDs, $\xi=10$ to 1000.

For the purposes of this calculation, we assume that the SQUID noise is low enough compared to the TES noise when $S_{I(\rm SQUID)}$ is less than half of $S_{I(\rm TES)}$, leading to a maximum degradation in energy resolution of about 20%. If a smaller degradation is required, the calculation can be adjusted accordingly. Using Eqs. (12) (in the zero frequency limit) and (13) and setting $S_{I(\rm SQUID)}<S_{I(\rm TES)}/2$ yields a minimum SQUID-input-coil inductance of

$$L_{\rm SQUID}>\frac{2\xi hR_0}{nk_BT_0}. \tag{14}$$

If the SQUID is placed close to the TES (perhaps even on the same chip), the inductance of the input coil can be large compared to any stray inductance, and the electrical response time of a properly matched circuit is $L_{\rm SQUID}/R_0>2\xi h/nk_BT_0$.

Combining the amplifier noise constraint of Eq. (14) and the condition for electrical stability, we estimate the achievable response time for a critically-damped TES microcalorimeter operated near the optimal energy resolution. In the strong-feedback limit, electrical stability requires a response time greater than 5.8 times the electrical time constant [Eq. (11)], so the amplifier noise restricts the response time of the SQUID to

$$\tau_{\rm eff}>\frac{11.6\xi h}{nk_BT_0}. \tag{15}$$

For example, assuming $\xi=500$, operation at $T=0.1$ K, and $n=4$, the critical-damping and noise-matching conditions lead to an electrical time constant $\tau_{\rm eff}>0.7\ \mu$s. A faster response time may be achieved by reducing the input inductance (or increasing the TES resistance) at the price of a loss in energy resolution. Alternatively, the use of SQUIDs with better energy sensitivity allows faster response times without sacrificing energy resolution. The noise matching and critical damping conditions are more stringent at lower temperatures. Thus, the required response time may place a limit on how cold a microcalorimeter can be operated, and hence on the energy resolution. Note that the limit that amplifier-noise and critical-damping conditions place on the response time is independent of both the resistance of the thermometer and the heat capacity of the detector.

VI. CRITICAL CURRENT

The critical current of the superconducting film also imposes a constraint on the response time of a TES microcalorimeter.[6] In this section, we discuss the complicated nature of the superconducting transition and introduce a two-fluid TES transition model to simplify calculations. We then develop an equation for the Ginzburg–Landau (GL)

critical current of a BCS superconductor in terms of appropriate parameters and use it to derive the critical-current constraint on the response time.

Usually, the theory of TES microcalorimeters assumes that the electrical resistance in the superconducting-normal transition region, $R \equiv V/I = R(T)$, is a simple function of the temperature. More generally, the resistance is a function of temperature, applied current, and applied magnetic field. The transition can be hysteretic. At low applied field and current, the form of the transition can be influenced by film nonuniformities, magnetic fields, and nonequilibrium effects near superconducting-normal interfaces. Even if the superconductor is perfectly uniform and negligible current is carried, the transition will have a nonzero width due to phase-slip events induced by thermal fluctuations.[14] At nonzero applied currents, a voltage can be generated across a uniform, isothermal superconducting film by a current flow near the critical current. In this section, we consider the last mechanism, and the limitation it imposes on the response time. While it may be desirable to fabricate a detector which is limited by other factors, such as geometric variations in transition temperature, the critical current still places a lower limit on the detector response time.

The form of a transition induced by a current flow near the critical current depends on whether the film is Type I (where the Ginzburg–Landau parameter $\kappa < 1/\sqrt{2}$) or Type II ($\kappa > 1/\sqrt{2}$). In a Type II superconducting film, the voltage is caused by the motion of quantized vortices. Unfortunately, in most Type II superconducting films (depending on correlations in the vortex lattice), vortex motion can introduce current noise larger than the Johnson noise in the film.[15,16] Further, since the Ginzburg–Landau parameter is inversely proportional to the mean-free path in dirty superconductors, thermometers with long mean-free paths and thus fast diffusion times will tend to be Type I. For the purposes of this discussion, the use of a Type I film is assumed.

In a Type I superconducting film, the voltage is produced by phase-slip events in one or more phase-slip lines (PSLs) in the film.[17,18] Depending on the geometry of the film, this regime may be characterized by steps in the $I-V$ curve associated with the nucleation of PSLs where the local critical current is exceeded. Operation in this regime can be complicated due to the presence of PSL-nucleation features. Apart from these steps, changes in the resistance of the film are caused by the temperature dependence of the critical current and, to a lesser extent, by the weak temperature dependence of the charge-imbalance relaxation length Λ_{Q*} in the superconducting film.[19] In the successful Skocpol–Beasley–Tinkham (SBT) model[17] of a Type I superconductor, the voltage is $V = 2N\rho_\lambda \Lambda_{Q*}(T)[I - \beta I_c(T)]$, where N is the number of PSLs, ρ_λ is the normal resistance per unit length of the superconductor, and $\beta \equiv \overline{I_c}/I_c$ is the ratio of the time-averaged critical current in the PSLs to the critical current of the film (typically about 0.5).

For the purposes of this calculation, we introduce a model which is similar in form to SBT, but simpler and more amenable to computation. The two-fluid transition model separates the sensor current $I(T)$ into a supercurrent component which is always some fraction c_I of the Ginzburg–Landau critical current of the film $I_c(T)$ and a quasiparticle component equal to the voltage V divided by some fraction c_R of the normal resistance R_n:

$$I(T) = c_I I_c(T) + V/(c_R R_n). \tag{16}$$

The two-fluid transition model neglects the weak temperature dependence of Λ_{Q*}, but otherwise reduces to the SBT model when $c_I = \beta$ and $c_R = 2N\rho_\lambda \Lambda_{Q*}/R_n$. Results derived with this formula should be roughly accurate and should have the proper scaling with respect to parameters such as the heat capacity and normal resistance.

In order to apply the two-fluid transition model, we must know the critical current of the sensor as a function of temperature. We assume that the sensor is a simple BCS superconductor that behaves in accordance with Ginzburg–Landau (GL) theory near the transition temperature. Combining the standard GL results for critical current density J_c with BCS results near T_c for critical field and effective penetration depth,[20] we arrive at the following expression for critical-current density:

$$J_c(T) \approx 1.54 \frac{H_c(0)}{\lambda_L(0)} \sqrt{\frac{l}{\xi_0}} (1-t)^{3/2}. \tag{17}$$

Here $H_c(0)$ is the BCS thermodynamic critical field at absolute zero, $\lambda_L(0)$ is the London penetration depth at zero temperature, l is the mean-free path in the film, ξ_0 is the BCS coherence length, and $t \equiv T/T_c$ is the reduced temperature.

The BCS critical field is[20]

$$\mu_0 H_c(0) = \Delta(0)\sqrt{\mu_0 N(0)}, \tag{18}$$

where $\Delta(0)$ is the energy gap parameter at zero temperature, μ_0 is the permeability of free space, and $N(0)$ is the density of states.

The London penetration depth is $\lambda_L(0) = \sqrt{m^*/(\mu_0 n_s^* e^{*2})}$, where m^* is the effective mass of the pair, n_s^* is the density of pairs, and e^* is the effective charge of the pair. We take $m^*/(n_s^* e^{*2}) \approx m_e/(n_e e^2)$, where m_e, n_e, and e are the corresponding free-electron mass, density, and charge. Using the London penetration depth, the BCS expression for coherence length $\xi_0 \approx 0.18\hbar v_f/k_B T_c$, the free-electron model for the normal-electron resistivity $\rho_N = m_e v_f/(n_e e^2 l)$, and Eqs. (17) and (18), we arrive at

$$J_c \approx 6.39(k_B T_c)^{3/2} \sqrt{\frac{N(0)}{\hbar \rho_N}} (1-t)^{3/2}. \tag{19}$$

This equation may be put in a more suggestive form by converting current density to current, resistivity to resistance, and using the free-electron expression for the heat capacity of the normal-electron system in the thermometer, $C_n = \pi^2 N(0) k_B^2 T_c U/3$, where U is the volume of the superconductor. Although experimental determinations of heat capacity vary from this expression by factors of 2 due to variations in effective mass, the scaling achieved by using this equation should be correct. Assuming that the film has a uniform cross section, we arrive at

$$I_c \approx 3.52 \sqrt{\frac{k_B C_n}{\hbar R_n}}\, T_c (1-t)^{3/2}, \tag{20}$$

an equation for the GL critical current I_c of a BCS superconductor free of microscopic parameters, in terms of the normal-electron heat capacity, normal electrical resistance R_n, transition temperature, and reduced temperature.

Achieving the theoretical critical current can be difficult. Ambient magnetic fields and nonuniformities of current flow in the film can significantly reduce the critical current. With careful magnetic shielding and the use of a superconducting ground plane to make the current flow uniform, however, approaching the theoretical GL critical current with a BCS superconducting film has been possible.[20]

We now estimate the response time using the calculated GL critical current and the two-fluid transition model. In the past, most theory has assumed that the electrical resistance is a simple function of temperature $R=R(T)$. In the two-fluid transition model, the resistance $R \equiv V/I$ is a function of voltage and current, and the current is a function of temperature and voltage $I=c_I I_c(T)+V/(c_R R_n)$. Therefore, at constant voltage bias, the resistance in the two-fluid transition model reduces to a simple function of temperature,

$$R=R(T)=\frac{V}{c_I I_c(T)+V/(c_R R_n)}, \tag{21}$$

and the theory previously developed for the detector response is valid. Note that it is difficult to directly compare the theoretical transition shape in Eq. (21) to experimental values since it is difficult to unambiguously determine the temperature of a voltage-biased sensor in the strong-feedback limit. However, preliminary comparison of detector performance in the critical-current limited regime agrees qualitatively with the predictions based on this formula.

The first step in estimating the response time in the two-fluid transition model is to calculate the logarithmic sensitivity of the film $\alpha \equiv d \log R / d \log T$ at constant voltage. Assuming that the film temperature is close to the transition temperature so that the GL critical current $I_c(T)=I_{c0}(1-T/T_c)^{3/2}$ applies, and using Eq. (21), we calculate

$$\alpha=\frac{3}{2}\left[\frac{(c_R R_n - R_0) c_I^2 I_{c0}^2 R_0}{c_R R_n P_0}\right]^{1/3}, \tag{22}$$

where R_0 is the resistance at equilibrium, and the equilibrium Joule Power, $P_0=V^2/R_0$, is a constant throughout the narrow transition in the strong-feedback limit. The detector is optimally biased when α is maximized (and the response time is minimized). Equation (22) is a maximum when $R_0 = c_R R_n/2$, so the maximum α is

$$\alpha_{\mathrm{MAX}}=\frac{3}{4}\left(\frac{2 c_I^2 I_{c0}^2 c_R R_n}{P_0}\right)^{1/3}. \tag{23}$$

Further, using $P_0=G T_0 \phi/n$, $T_0 \approx T_c$ in the strong-feedback limit, and Eq. (20) for I_{c0}, we calculate

$$\alpha_{\mathrm{MAX}} \approx 2.19 \left(\frac{k_b T_c}{\hbar}\frac{C_n}{G}\frac{n}{\phi}\, c_R c_I^2\right)^{1/3}. \tag{24}$$

High values of α_{MAX} can be achieved with reasonable detector parameters. For example, taking soft x-ray detector parameters $C_n=1$ pJ/K, $G=1$ nW/K, $T_c=0.1$ K, $\phi=1$, and $n=4$, and assuming $c_R=1$ and $c_I=0.5$, we compute $\alpha_{\mathrm{MAX}} \sim 500$.

From Eq. (1), when voltage biased, the effective time constant of the film is $\tau_{\mathrm{eff}} \approx \tau_0/(1+\alpha\phi/n)$, where $\tau_0=C_{\mathrm{tot}}/G$ is the natural time constant of the microcalorimeter and C_{tot} is the total heat capacity of the thermometer and absorber. Just below the transition, BCS superconductors have a heat capacity approximately 2.43 times the value of the normal metal.[20] Using Eq. (24), we arrive at an expression for the minimum effective time constant of a TES microcalorimeter in the strong-feedback, critical-current-limited two-fluid transition model:

$$\tau_{\mathrm{MIN}} \approx 0.46 \left(\tau_0^2 \frac{C_{\mathrm{tot}}}{C_n}\frac{\hbar}{k_B T_c}\frac{n^2}{\phi^2}\frac{1}{c_R c_I^2}\right)^{1/3}. \tag{25}$$

There are several important observations to make about Eq. (25). First, the minimum time constant τ_{MIN} is independent of the electrical resistance. Second, the minimum response time due to the critical current is independent of the saturation energy. The saturation energy of the microcalorimeter is proportional to $C_{\mathrm{tot}} T_c/\alpha$. But the logarithmic sensitivity α and the minimum response time of the sensor are both independent of the total heat capacity as long as the natural time constant is held fixed (by increasing G as C_{tot} is increased) and the ratio C_{tot}/C_n is held fixed (by increasing the size of the thermometer by the same ratio as the size of the absorber.) Since Eq. (25) is derived in the strong-feedback limit, it should be noted that these observations are only valid for $\alpha_{\mathrm{MAX}} \gg n/\phi$.

The two-fluid transition model critical current is consistent with 1 μs microcalorimeter response time for a BCS superconductor in the strong-feedback, GL limit. For example, choosing $\tau_0=25$ μs, $C_{\mathrm{tot}}/C_n=4$, $T_c=0.1$ K, $\phi=1$, $n=4$, $c_R=1$, and $c_I=0.5$, from Eq. (24) $\alpha_{\mathrm{MAX}} \sim 100$ and from Eq. (25) $\tau_{\mathrm{MIN}} \sim 1$ μs.

VII. SELF HEATING

Joule heating in a transition-edge thermometer can cause a temperature variation across the film, reducing the sensitivity of the thermometer and affecting the response time. The effects of self heating are dependent on the thermal conductances and geometry of the detector and the form of the superconducting transition. In the general case the effects of self heating must be analyzed numerically. Instead of a detailed analysis of a specific geometry, we present a simple calculation to show how self-heating constraints scale and to demonstrate that self heating does not place a fundamental limit on the response time.

For the purposes of this analysis, we model a transition-edge sensor as two isothermal elements at different temperatures connected by a thermal conductance. Near 100 mK, this thermal conductance is dominated by the Wiedemann–Franz thermal conductance of the normal electrons, $G_{\mathrm{WF}}=L_n T/R_n$, where the Lorentz number $L_n \approx 24.5$ nW Ω K^{-2}. We make the worst-case assumption that

the entire detector bias power P is dissipated in the first element, flows to the second element, and then escapes through a thermal conductance G to a heat bath. The temperature drop across the film is then $\Delta T = P/G_{\mathrm{WF}}$.

The characteristic transition width ΔT_{char} associated with a logarithmic sensitivity α is of order $\Delta T_{\mathrm{char}} \sim T_c/\alpha$. As a rough estimate, self heating will not limit detector performance when the temperature drop associated with self heating is smaller than this characteristic transition width, or when $P/G_{\mathrm{WF}} < T_c/\alpha$. When the heat bath is cooled to well below the equilibrium sensor temperature, the equilibrium power $P \approx G T_0/n$, so to avoid self heating we require $G_{\mathrm{WF}} > G\alpha/n$. Substituting $G_{\mathrm{WF}} = L_n T/R_n$, we calculate

$$R_n < \frac{L_n T_0}{G}\left(\frac{n}{\alpha}\right). \tag{26}$$

Thus, if the normal resistance of the TES is low enough, self heating is negligible. For instance, consider a typical x-ray microcalorimeter with $G = 1$ nW/K, $T_0 = 0.1$ K, $n = 4$, and $\alpha = 100$. Then, from Eq. (26), self heating should not be important as long as $R_n < 0.1\Omega$.

The self heating in a TES is unimportant if its normal resistance is low enough. Since the limits on the response time due to electrical stability, amplifier noise, and critical current are independent of the resistance (as long as the inductance scales accordingly), in principle the resistance can always be lowered enough to make self heating negligible.

VIII. CONCLUSIONS

In order to optimize a TES microcalorimeter for a specific application, careful detector design is required. In the TES microcalorimeter, the response time limitations we have discussed couple the optimization for response time and energy resolution. Different applications will impose different optimization constraints, including the required absorber area and quantum efficiency, the available heat bath temperature, the required response time, and the maximum photon energy. Since the constraints on optimization are different for each application, the development of a general optimization procedure is difficult. Instead, we present here a description of two possible detectors for use at different energies.

A number of detector design rules are suggested by the above calculations and are listed here.

(1) The diffusion times in the absorber should be made much shorter than the response time, or pulse shape variations should be corrected in the data analysis.

(2) Negative electrothermal feedback should be used so that the response time can be close to the diffusion time in the thermometer without sacrificing energy resolution.

(3) The response time should be sufficiently longer than the L/R time constant for stable electrical operation [Eqs. (10) and (11)].

(4) Stray inductances should be kept small compared to the SQUID input inductance.

(5) The SQUID must be quiet. The SQUID input inductance must be chosen so that the SQUID noise is smaller than the TES noise [Eq. (14)].

(6) The critical current of the sensor should be maximized. A large volume superconducting film should be used; the heat capacity of the sensor should be a significant fraction of the total microcalorimeter heat capacity [Eq. (25)]. Careful magnetic shielding and the use of a superconducting ground plane may be necessary.

(7) Self heating should be minimized by keeping the normal resistance of the TES small [Eq. (26)].

Although the minimum response time is in principle independent of the saturation energy, achieving response times near the limits described in this article is considerably easier for detectors designed for low-energy photons. Consider for example a detector designed for response times of 1 μs for optical to near-infrared photons of order 1 eV. In such a detector, the TES itself could be used as the absorber. Assuming the TES is a BCS superconductor and the response time is critical-current limited, if the natural time constant is $\tau_0 = 35$ μs, $C_{\mathrm{TOT}}/C_{n(\mathrm{TES})} \approx 2.43$ (since the heat capacity of a BCS superconductor is about 2.43 times higher than a normal metal), $T_c = 0.1$ K, $\phi = 1$, $n = 4$, $c_R = 1$, and $c_I = 0.5$, from Eq. (24) $\alpha_{\mathrm{MAX}} \sim 125$ and from Eq. (25) $\tau_{\mathrm{MIN}} \sim 1$ μs. In order to saturate near 1 eV, the heat capacity should be $C_{\mathrm{TOT}} > E\alpha_{\mathrm{MAX}}/T_c = 0.2$ fJ/K. We will take $C_{\mathrm{TOT}} = 0.5$ fJ/K. Then to maintain a natural time constant of 35 μs the thermal conductance should be $G \sim 14$ pW/K. In a sensor this small, diffusion times are fast enough to never be a limiting factor. To avoid self heating, the normal resistance of the TES should be $R_n < L_n T_0 n/(G\alpha) \sim 5\Omega$ [Eq. (26)]. We will choose a normal resistance of 4 Ω. To avoid electrical instability, the electrical response time L/R must be faster than 170 ns [Eq. (11)]. Assuming a bias resistance of $R_0 = 2$ Ω, a total circuit inductance of less than about 340 nH is required, which is easily achieved. Note that if a SQUID with $\xi = 500$ is used, the above detector-design procedure will automatically satisfy the amplifier noise condition [Eq. (15)] as long as the response time is slower than about 700 ns. The fundamental limit on the energy resolution of such a detector would be[2] about 30 meV FWHM.

A similar procedure can be used to design a detector for response times of 10 μs for 6.4 keV Fe $K\alpha$ x rays. Then choosing $\tau_0 = 700$ μs, $C_{\mathrm{TOT}}/C_{n(\mathrm{TES})} \sim 5$, $T_c = 0.1$ K, $\phi = 1$, $n = 4$, $c_R = 1$, and $c_I = 0.5$, from Eq. (24) $\alpha_{\mathrm{MAX}} \sim 270$ and from Eq. (25) $\tau_{\mathrm{MIN}} \sim 10$ μs. The heat capacity should be $C_{\mathrm{TOT}} > E\alpha_{\mathrm{MAX}}/T_c = 2.7$ pJ/K. We will take $C_{\mathrm{TOT}} = 4$ pJ/K. Then to maintain a natural time constant of 700 μs the thermal conductance should be $G \sim 6$ nW/K. In this detector the absorber heat capacity would correspond to a gold film 250 μm$\times$250 μm$\times$5 μm. The expected mean-free path in a high quality deposited film this thick would be ~ 2 μm, leading to a diffusion time in the absorber of order 22 ns, which is short enough as compared to 10 μs that pulse shape variations will be small. To avoid self heating, the normal resistance of the TES should be $R_n < L_n T_0 n/(G\alpha) < 6.4$ mΩ. We will choose a normal resistance of 4 mΩ. To avoid electrical instability, the electrical response time L/R must be faster than 1.7 μs [Eq. (11)]. Assuming a bias resistance of $R_0 = 2$ mΩ, a total circuit inductance of less than about 3.4 nH is required. The amplifier-noise criterion is the same as in the optical detector above, satisfied as long as τ

$>$700 ns. A normal sheet resistance of 4 mΩ corresponds to a TES film thickness of $\sim 1\ \mu$m, and dimensions of about 300 μm$\times$300 μm. A typical mean-free path in a TES of these dimensions would be of order 150 nm, corresponding to a TES diffusion time of about 0.4 μs, which is fast enough for a 10 μs detector. The fundamental limit on the energy resolution of such a detector would be[2] about 1.6 eV FWHM.

The fabrication of fast-response-time detectors for operation at higher energies will be challenging. For the example above (a 6.4 keV x-ray detector), the stray inductance must to be small compared to the 3.4 nH SQUID input inductance. In this case, it might be necessary to fabricate x-ray detectors with on-chip SQUID preamplifiers or transformers. The use of superconducting ground planes might also be necessary to reduce the inductance of the lines between the detector and the SQUID. While it is in principle possible to make 6.4 keV detectors which operate at 1 μs, the total circuit inductance would then have to be less than 100 pH, which would be very difficult to achieve.

We have shown that diffusion times, electrical stability, amplifier noise, and critical current place a lower limit on the response time of a TES microcalorimeter. These constraints are more stringent at lower temperatures. The response-time limit is of order 1 μs for a BCS superconductor operating at 0.1 K. The response time must be slower when operated near optimal resolution at temperatures below 0.1 K. With proper design and large enough Wiedemann–Franz thermal conductances, self heating does not limit the response time. The response time is independent of the intended photon energy for fixed natural time constant, but constraints on the total circuit inductance become more difficult for fast response at higher energies.

As previously mentioned, a response time of 8 μs has been demonstrated for a low-heat-capacity microcalorimeter appropriate for optical photons.[6] The response time of this device was limited by instabilities due to a total circuit inductance which was too large. A response time of about 200 μs has been demonstrated for a 6 keV x-ray detector, which was limited by a slow natural time constant and a film critical current which was significantly worse than the theoretical GL value for a BCS superconductor.[5] It should be possible to fabricate detectors with considerably faster response times than has been achieved to date.

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